



DØ B_s^0 Oscillation Combination for Summer 2007

The DØ Collaboration

URL <http://www-d0.fnal.gov>

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We describe the combination of DØ B_s^0 – \bar{B}_s^0 oscillation frequency measurements prepared for the Heavy Flavors Averaging Group (HFAG) world average and based on preliminary results presented at the summer 2007 conferences. Using $\sim 2.4 \text{ fb}^{-1}$ of data in the $B_s \rightarrow \mu\nu D_s(\phi\pi)X$, $B_s \rightarrow \mu\nu D_s(K^*K)X$, $B_s \rightarrow e\nu D_s(\phi\pi)X$, $B_s \rightarrow \pi D_s(\phi\pi)X$ channels, as well as $\sim 1.2 \text{ fb}^{-1}$ in the $B_s \rightarrow \mu\nu D_s(K_SK)X$ channel, we find $\Delta m_s = 18.53 \pm 0.93(\text{stat}) \pm 0.30(\text{syst}) \text{ ps}^{-1}$, with an estimated total significance of 2.9σ . This result supersedes that quoted in DØ Note 5474.

Preliminary Results from Summer 2007 Conferences for HFAG World Average

I. INTRODUCTION

This note describes the combination of DØ B_s oscillation measurements for use in the HFAG Oscillations combination [1] prepared for Summer 2007 results. Details of the analyses used in this note are given in [2, 3] and are not repeated here. However, minor changes with respect to the information presented in [2, 3] are summarized.

Modes used in the combination are described in Table I, which also includes the data used for each mode.

TABLE I: Modes used in the B_s oscillations combination, the names by which they will be referred in this note, the data samples analyzed, and references to more detailed information.

Decay Mode	Name	RunIIa (fb^{-1})	RunIIb (fb^{-1})	Ref.
$B_s \rightarrow e\nu D_s(\phi\pi)X$	$e\phi\pi$	1.3	1.1	[2]
$B_s \rightarrow \mu\nu D_s(K^*K)X$	μK^*K	1.3	1.1	[2]
$B_s \rightarrow \mu\nu D_s(\phi\pi)X$	$\mu\phi\pi$	1.3	1.1	[2]
$B_s \rightarrow \pi D_s(\phi\pi)X$	$\pi\phi\pi$		2.4	[2]
$B_s \rightarrow \mu\nu D_s(K_SK)X$	μK_SK	1.2	—	[3]

II. COMBINATION

The combined DØ result was obtained in two steps: first the individual amplitude scans for each of the modes considered were combined, and then this combined amplitude scan was translated to a likelihood curve from which the central value and errors of our measured Δm_s were extracted.

A. Amplitude Scan Combinations

Individual amplitude scans vs. probe Δm (with RunIIa and RunIIb results considered separately) were combined using the **COMBOS** program [4]. The individual scans, including statistical and systematic errors are shown in Fig. 1 and, in abbreviated form, in tables in Appendix A. All data, as well as a full set of intermediate results, can be found at [5]. These scans represent the final versions of each channel's preliminary analysis.

A list of all systematic errors considered, along with which modes included estimates of each error is given in Table II. Systematic errors assumed to be 100% correlated in the combination, are summarized in Table III.

The final, combined scan, is shown in Fig. 2. Sensitivities for each mode at the 95% C. L. were defined as the probe Δm value at which:

$$1.645 \sigma_{\mathcal{A}}(\Delta m_{\text{sens}}) = 1$$

and are listed in Table IV.

1. Differences with respect to summer results

Scans for individual modes used in this combination are identical to those presented in [2, 3]. However, the combination of amplitudes done here has the following differences from that done in [2].

1. The μK_SK mode is included here, but not in [2].
2. Several small sources of systematic were mistakenly left out of the combination in [2]. They are included here.
3. Several typos in the **COMBOS** driver files for correlations between systematics were corrected.
4. Full details on the differences between this combination and that of [2] are given in [5].

None of these minor changes had a significant effect on the result as the sensitivities of the combination in [2] and that presented here are the same – 27.3 ps^{-1} .

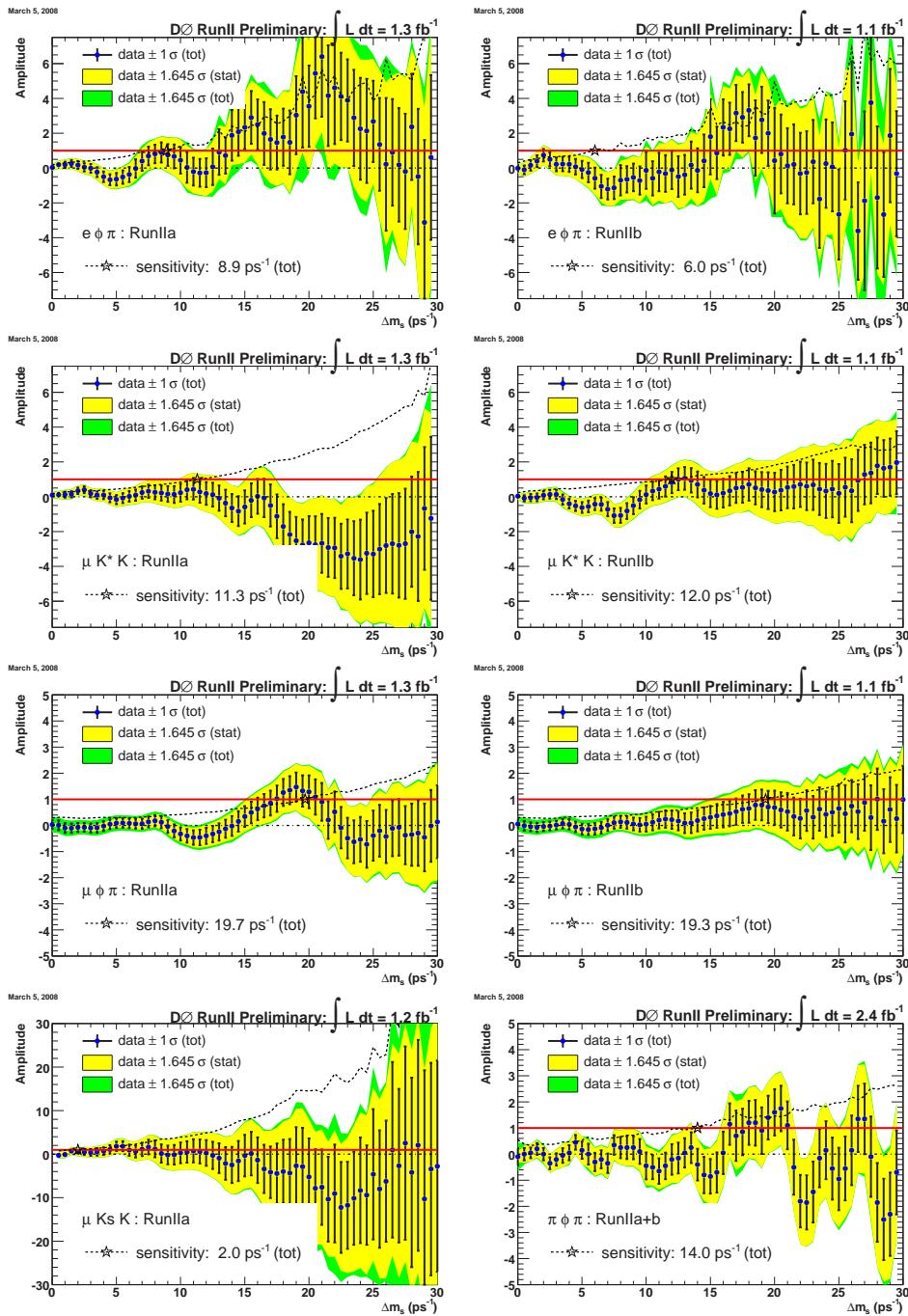


FIG. 1: Amplitude scans for the individual analysis modes.

B. Likelihood and Extraction of Δm_s

The combined amplitude scan was translated to a likelihood using the average relationship between the fitted amplitude at a scan point Δm , its error, and the difference between the log likelihood at that point and the log likelihood at infinity [6]:

$$\Delta \ln \mathcal{L} = -\ln \mathcal{L}(\Delta m) + \ln \mathcal{L}(\infty) = \frac{\frac{1}{2} - \mathcal{A}_{\text{fit}}(\Delta m)}{\sigma_{\mathcal{A}}^2}$$

This was done twice: once using the statistical error on \mathcal{A}_{fit} as $\sigma_{\mathcal{A}}$ and once using the total (statistical + systematic)

TABLE II: All sources of systematic error considered in the analyses. Entries marked with a “✓” indicate that a numerical estimate of the systematic in question was made for that mode, while blank entries indicate contributions that were estimated to be small.

Description	Name	$e\phi\pi$ IIa IIb	μK^*K IIa IIb	$\mu\phi\pi$ IIa IIb	$\pi\phi\pi$ IIa+b	μK_SK IIa
Sample Composition						
$D_s D_s$ contrib. uncertainty	$B_s^0 \rightarrow D_s D_s = 4.7\%$	✓ ✓	✓ ✓	✓ ✓		✓
use $D_s D_s$ PDG value	$D_s D_s$ PDG					✓
$D_s \mu$ contrib. uncertainty	$B_s^0 \rightarrow D_s \mu \nu X = 5.5\%$	✓ ✓	✓ ✓	✓ ✓		✓
$D_s D$ contrib. uncertainty	$B_s^0 \rightarrow D_s D$			✓ ✓		
Muon p_T	$p_T^\mu > 6$ GeV	✓ ✓	✓ ✓	✓ ✓		✓
Sample Composition						
B_s lifetime uncertainty	$c\tau_{B_s}$	✓ ✓	✓ ✓	✓ ✓		✓
use PDG B_s lifetime	$c\tau_{B_s} = 438\mu\text{m}$			✓ ✓		✓
Backgrounds						
Bkg fraction varied by 20%	\mathcal{F}_{bkg}				✓	
Mass Bkg shape	M_{bkg} shape					✓
Peaking bkg in signal	$\mathcal{F}_{peak(cc)-sig}$	✓ ✓	✓ ✓	✓ ✓		✓
Prompt fraction in bkg	\mathcal{F}^0	✓ ✓		✓ ✓		
Prompt bkg resolution SF	$s f_{bkg}$	✓ ✓	✓ ✓	✓ ✓		✓
Peaking (quasi-V) frac. in bkg	$\mathcal{F}_{peak-bkg}$	✓ ✓		✓ ✓		✓
Peaking width in bkg	$\sigma_{peak-bkg}$	✓ ✓		✓ ✓		
Long-lived bkg	$c\tau_{bkg}$	✓ ✓		✓ ✓		
Negative lifetime bkg	$c\tau_{neg}$					✓
Frac. of tagging sensitive bkg	\mathcal{F}_{tsens}	✓ ✓		✓ ✓		✓
Frac. of oscillating bkg	\mathcal{F}_{osc}	✓ ✓		✓ ✓		✓
Number of D_s candidates	ΔN_{D_s}	✓ ✓		✓ ✓		✓
Number of reflection candidates	ΔN_{refl}		✓ ✓	✓ ✓		✓
K-Factors						
K-Factor resolution	K-fact var. 2%	✓ ✓	✓ ✓	✓ ✓	✓	
K-Factor smoothing	K-fact smooth		✓ ✓	✓ ✓		✓
Use reco K-factor	reco K-fact					✓
K-factor p_T dependence	K-fact p_T					✓
Dilution						
OST dilution uncertainty	dilution	✓ ✓	✓ ✓	✓ ✓	✓	✓
VPDL Resolution						
Resolution SF in signal	$s f_{sig}$	✓ ✓		✓ ✓	✓	✓
Frac. of 1st Gaussian in signal	\mathcal{F}_{G1}	✓ ✓				
Efficiency vs VPDL						
Efficiency vs VPDL	ϵ vs. VPDL			✓ ✓	✓	
Other						
Non-zero $\Delta\Gamma/\Gamma$	$\Delta\Gamma/\Gamma$			✓ ✓		
Cabbibo Ratio	Cabbibo Ratio					✓

error. These scans are shown in Fig. 3.

As the shapes of the likelihood curves near their minima are quite parabolic, our measured value of Δm_s and its errors were derived by fitting a quadratic function to the likelihood scan. An estimate of the systematic error on the measurement was obtained by subtracting, in quadrature, the error derived from the statistical-error-only likelihood curve from that of the total-error curve. The result is:

$$\Delta m_s = 18.53 \pm 0.93(\text{stat}) \pm 0.30(\text{syst}) \text{ ps}^{-1}.$$

One measure of the significance of this result is given by:

$$\text{Significance } (\sigma) = \sqrt{-2\Delta \ln \mathcal{L}}.$$

Using this definition, we obtained a statistical significance of 3.0σ and a total significance (including systematic errors) of 2.9σ .

TABLE III: Correlations among systematic errors in the individual analysis modes. For a given systematic, those entries marked with a “✓” were assumed to be 100% correlated in the combination. Those entries marked with “—” indicate existing systematics for which no correlation is assumed, while blank entries correspond to a negligible systematic estimated for that mode.

Description	Name	$e\phi\pi$ IIa	$e\phi\pi$ IIb	μK^*K IIa	μK^*K IIb	$\mu\phi\pi$ IIa	$\mu\phi\pi$ IIb	$\pi\phi\pi$ IIa+b	μK_SK IIa
$D_s D_s$ contrib. uncertainty	$B_s^0 \rightarrow D_s D_s = 4.7\%$	✓	✓	✓	✓	✓	✓		✓
$D_s \mu$ contrib. uncertainty	$B_s^0 \rightarrow D_s \mu \nu X = 5.5\%$	✓	✓	✓	✓	✓	✓		✓
$D_s D$ contrib. uncertainty	$B_s^0 \rightarrow D_s D$					✓	✓		
use PDG B_s lifetime	$c\tau_{B_s} = 438\mu\text{m}$					✓	✓		✓
OST dilution uncertainty ^a	dilution	✓	✓	✓	✓	✓	✓	✓	✓
Resolution SF in signal ^b	$s f_{sig}$	✓	—			—	—	—	✓
Frac. of 1st Gaussian in signal	\mathcal{F}_{G1}	—	—						
Non-zero $\Delta\Gamma/\Gamma$	$\Delta\Gamma/\Gamma$					✓	✓		

^aDilution systematics were considered correlated among all channels. (All channels used the same variation for the OST.) Since the $\mu\phi\pi$ channel also used the SST its correlation with the others is large, but not 100%. We approximated it with 100%.

^bThe RunIIa VPDL resolution systematic was considered to be 100% correlated between the $e\phi\pi$ and μK_SK channels, which both used an average resolution scale factor derived from the same RunIIa J/ψ data set. Note that no resolution systematic was quoted for the μK^*K channel, while the RunIIa $\mu\phi\pi$ channel used event-by-event scale factors.

Also note that resolutions systematics were derived independently for the $\pi\phi\pi$ channel and hence were uncorrelated.

The RunIIb VPDL resolution systematic was considered to be 100% correlated between the $e\phi\pi$ and $\mu\phi\pi$ channels, which both used an average resolution scale factor derived from the same RunIIb J/ψ data set.

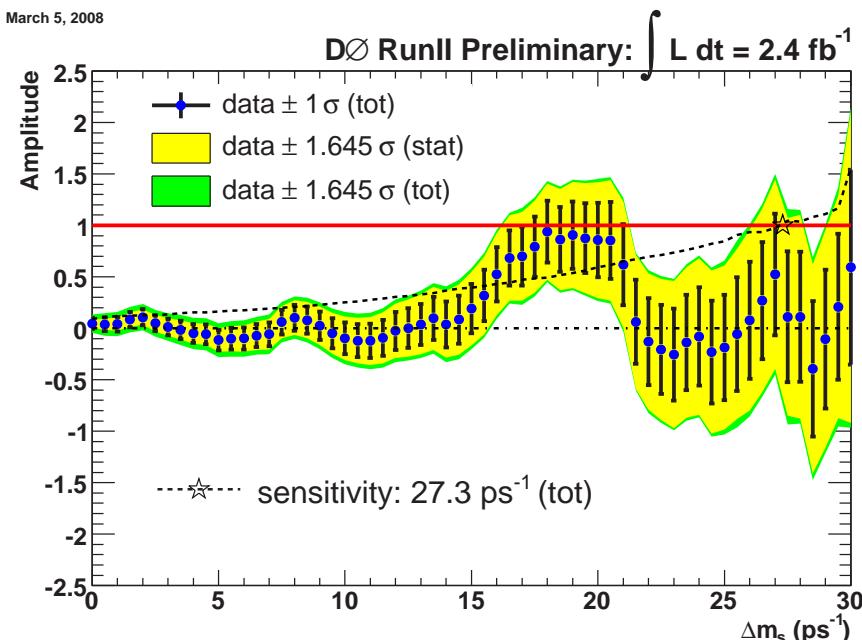


FIG. 2: The combined amplitude scan.

TABLE IV: Sensitivities at the 95% C. L., defined from the amplitude scans for each analysis mode separately and for the combined result.

Decay Mode	95% C.L. Sensitivity (ps ⁻¹)		
	RunIIa	RunIIb	RunIIa+b
$e\phi\pi$	8.9	6.0	10.0
μK^*K	11.3	12.0	15.2
$\mu\phi\pi$	19.7	19.3	23.9
μK_SK	2.0		
Semi-Leptonic	20.5	21.5	25.4
$\pi\phi\pi$			14.0
Full Combination			27.3

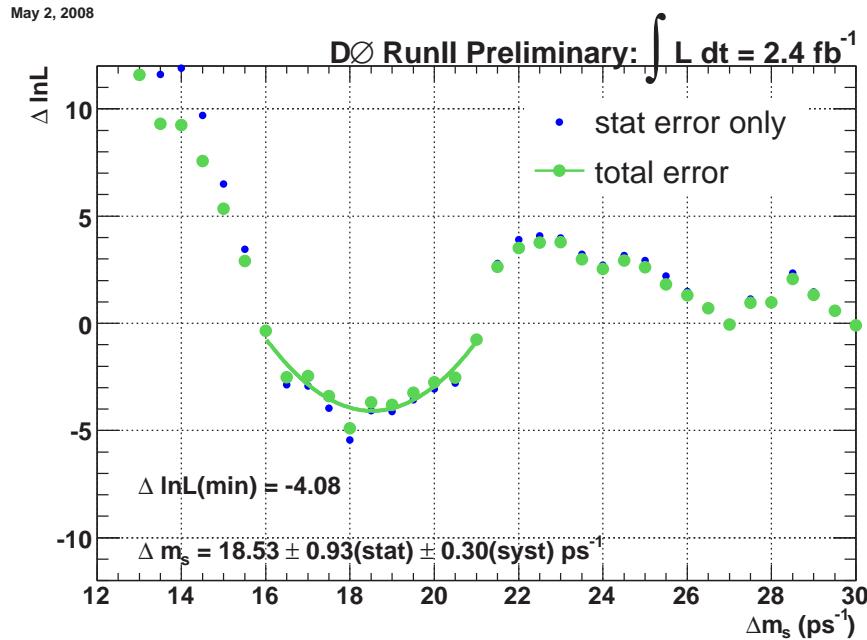


FIG. 3: The combined likelihood scan ($\Delta \ln \mathcal{L}$ vs. Δm) derived from the combined amplitude fit values. Values of $\Delta \ln \mathcal{L}$ using both statistical errors on the fitted amplitude values (small points) and using total errors (large points) are shown as is the parabolic fit to the total error $\Delta \ln \mathcal{L}$ scan, which is used to derive the measured value of Δm_s .

1. Differences with respect to summer results

The same method of extracting the measured value of Δm_s from the combined amplitude scan was employed as in [2] (although different code was used). Differences in the details of this extraction are given below.

1. The final result differs slightly between the two results:
 $\Delta m_s = 18.52 \pm 0.91(\text{stat})$ ([2], Fig. 15)
 $\Delta m_s = 18.53 \pm 0.93(\text{stat})$ (this note)
2. The statistical significance quoted in [2] was 3.1σ , while that calculated here is 3.0σ .
3. A systematic error was estimated on Δm_s in this note, using the method outline above, while no systematic on Δm_s was quoted in [2].
4. The total significance of the result found here is 2.9σ . No total significance is quoted in [2].

III. SUMMARY

We have presented the method used to combine searches for B_s oscillations at DØ. Using $B_s \rightarrow e\nu D_s(\phi\pi)X$, $B_s \rightarrow \mu\nu D_s(K^*K)X$, $B_s \rightarrow \mu\nu D_s(\phi\pi)X$, and $B_s \rightarrow \pi D_s(\phi\pi)X$ candidates in $\sim 2.4 \text{ fb}^{-1}$ of data and $B_s \rightarrow \mu\nu D_s(K_SK)X$ candidates in $\sim 1.2 \text{ fb}^{-1}$ of data, we find a preliminary value for the B_s oscillation frequency:

$$\Delta m_s = 18.53 \pm 0.93(\text{stat}) \pm 0.30(\text{syst}) \text{ ps}^{-1}.$$

This corresponds to a significance of $\sim 2.9\sigma$, calculated from the depth of the $\Delta \ln \mathcal{L}$ curve at the minimum.

- [1] see: <http://www.slac.stanford.edu/xorg/hfag/osc/index.html>
- [2] The DØ Collaboration, “Measurement of the Flavor Oscillation Frequency of B_s^0 Mesons at DØ”, DØ Note 5474, Aug. 2007.
- [3] The DØ Collaboration, “A Search for B_s^0 Oscillations Using $B_s^0 \rightarrow D_s \mu X (D_s \rightarrow K_S^0 K)$ Decays”, DØ Note 5254, Oct. 2006.
- [4] COMBOS ver. 3.22, <http://lepbosc.web.cern.ch/LEPBOSC/combos/>
COMBOS manual: <http://www.slac.stanford.edu/xorg/hfag/docs/>
- [5] <http://hep.physics.indiana.edu/~hgevans/analysis/hfag/>
- [6] H.G. Moser and A. Roussarie, Nucl. Instrum. and Methods **A384**, 491 (1997).

APPENDIX A: AMPLITUDE SCAN DATA

The following tables show a breakdown of the systematic errors in each analysis mode at representative Δm scan points. Systematics for all Δm scan points as well as the values of $\Delta \mathcal{A}$ and $\Delta \sigma$ used to compute each systematic are given in [5].

TABLE V: Breakdown of systematic errors for the RunIIa $e\phi\pi$ mode.

Osc. frequency	ps^{-1}	0.0	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0
Fitted Ampl.	\mathcal{A}	+0.036	-0.016	-0.371	+0.752	-0.267	+2.345	+1.776	+6.401	+2.255	+0.189
$c\tau_{Bs}$		0.007	0.011	0.010	0.014	0.016	0.024	0.059	0.021	0.006	0.086
\mathcal{F}_{peak_sig}		0.008	0.022	0.015	0.020	0.040	0.022	0.131	0.042	0.120	0.234
$s f_{bkg}$		0.002	0.005	0.003	0.004	0.002	0.001	0.004	0.013	0.002	0.008
\mathcal{F}_{peak_bkg}		0.003	0.009	0.007	0.004	0.007	0.016	0.003	0.014	0.009	0.002
\mathcal{F}^0		0.005	0.012	0.009	0.009	0.019	0.009	0.027	0.009	0.008	0.016
$c\tau_{bkg}$		0.000	0.011	0.009	0.011	0.007	0.000	0.005	0.058	0.032	0.004
σ_{peak_bkg}		0.001	0.003	0.002	0.001	0.003	0.001	0.004	0.010	0.004	0.004
\mathcal{F}_{tsens}		0.022	0.005	0.003	0.005	0.005	0.004	0.024	0.011	0.002	0.052
\mathcal{F}_{osc}		0.034	0.020	0.015	0.005	0.010	0.036	0.014	0.011	0.015	0.014
ΔN_{Ds}		0.012	0.026	0.015	0.024	0.016	0.019	0.027	0.068	0.010	0.025
K-fact var. 2%		0.006	0.017	0.163	0.113	0.121	0.040	0.207	2.210	1.641	0.440
dilution		0.003	0.004	0.003	0.005	0.003	0.004	0.005	0.013	0.002	0.005
$B_s^0 \rightarrow D_s D_s = 4.7\%$		0.028	0.031	0.028	0.031	0.028	0.029	0.031	0.038	0.025	0.025
$B_s^0 \rightarrow D_s \mu\nu X = 5.5\%$		0.014	0.006	0.007	0.007	0.011	0.008	0.005	0.019	0.024	0.019
$p_T^\mu > 6 \text{ GeV}$		0.013	0.007	0.006	0.006	0.018	0.003	0.003	0.044	0.035	0.050
$s f_{sig}$		0.002	0.001	0.017	0.024	0.026	0.032	0.010	0.015	0.048	0.022
\mathcal{F}_{G1}		0.004	0.033	0.078	0.119	0.147	0.221	0.184	0.262	0.158	0.180
Total Syst	σ_{syst}	0.056	0.067	0.186	0.173	0.203	0.236	0.317	2.228	1.655	0.544
Stat. uncertainty	σ_{stat}	0.100	0.245	0.389	0.604	0.809	1.232	1.609	2.639	2.418	3.213
Total	σ_{tot}	0.114	0.254	0.431	0.628	0.834	1.254	1.640	3.454	2.930	3.259

TABLE VI: Breakdown of systematic errors for the RunIIb $e\phi\pi$ mode.

Osc. frequency	ps^{-1}	0.0	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0
Fitted Ampl.	\mathcal{A}	-0.011	+0.217	-0.579	-0.532	-0.110	+1.801	+3.315	+0.137	+0.258	-1.879
$c\tau_{Bs}$		0.000	0.002	0.026	0.042	0.050	0.127	0.055	0.043	0.085	0.026
\mathcal{F}_{peak_sig}		0.025	0.043	0.057	0.052	0.038	0.044	0.093	0.089	0.036	0.095
$s f_{bkg}$		0.004	0.001	0.004	0.004	0.001	0.011	0.025	0.031	0.030	0.017
\mathcal{F}_{peak_bkg}		0.003	0.005	0.006	0.004	0.003	0.002	0.002	0.005	0.026	0.014
\mathcal{F}^0		0.003	0.006	0.007	0.002	0.003	0.016	0.029	0.014	0.023	0.002
$c\tau_{bkg}$		0.002	0.001	0.011	0.014	0.014	0.041	0.018	0.007	0.028	0.007
σ_{peak_bkg}		0.002	0.001	0.002	0.004	0.001	0.001	0.016	0.010	0.009	0.006
\mathcal{F}_{tsens}		0.045	0.003	0.000	0.002	0.003	0.010	0.014	0.048	0.044	0.020
\mathcal{F}_{osc}		0.052	0.037	0.017	0.011	0.015	0.003	0.006	0.003	0.017	0.010
ΔN_{Ds}		0.017	0.028	0.032	0.017	0.036	0.084	0.111	0.090	0.108	0.094
K-fact var. 2%		0.000	0.028	0.171	0.232	0.204	1.567	0.404	0.639	0.269	4.483
dilution		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$B_s^0 \rightarrow D_s D_s = 4.7\%$		0.025	0.027	0.026	0.024	0.024	0.022	0.023	0.023	0.024	0.030
$B_s^0 \rightarrow D_s \mu\nu X = 5.5\%$		0.017	0.011	0.011	0.009	0.013	0.009	0.016	0.015	0.020	0.009
$p_T^\mu > 6 \text{ GeV}$		0.016	0.016	0.012	0.007	0.005	0.021	0.014	0.014	0.040	0.008
$s f_{sig}$		0.001	0.006	0.016	0.009	0.013	0.006	0.045	0.110	0.081	0.063
\mathcal{F}_{G1}		0.003	0.038	0.102	0.043	0.062	0.174	0.245	0.322	0.225	0.132
Total Syst	σ_{syst}	0.083	0.087	0.215	0.248	0.228	1.586	0.502	0.740	0.397	4.488
Stat. uncertainty	σ_{stat}	0.169	0.385	0.567	0.754	1.070	1.476	1.404	2.028	2.504	2.486
Total	σ_{tot}	0.189	0.395	0.606	0.794	1.094	2.167	1.491	2.158	2.535	5.130

TABLE VII: Breakdown of systematic errors for the RunIIa $\mu K^* K$ mode.

Osc. frequency	ps^{-1}	0.0	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0
Fitted Ampl.	\mathcal{A}	+0.097	+0.179	+0.026	+0.162	+0.190	-0.582	-1.707	-2.672	-3.606	-2.794
$s f_{bkg}$		0.000	0.001	0.003	0.005	0.005	0.004	0.002	0.000	0.002	0.003
$B_s^0 \rightarrow D_s D_s = 4.7\%$		0.017	0.014	0.018	0.015	0.017	0.027	0.025	0.025	0.017	0.012
$B_s^0 \rightarrow D_s \mu \nu X = 5.5\%$		0.033	0.006	0.013	0.010	0.011	0.021	0.019	0.000	0.006	0.014
ΔN_{refl}		0.001	0.004	0.002	0.003	0.004	0.005	0.012	0.021	0.026	0.028
K-factor var. 2%		0.011	0.017	0.025	0.005	0.012	0.313	0.265	0.154	0.546	0.261
$c\tau_{B_s}$		0.013	0.007	0.042	0.040	0.050	0.062	0.080	0.110	0.019	0.061
$p_T^\mu > 6 \text{ GeV}$		0.019	0.006	0.011	0.006	0.006	0.012	0.012	0.004	0.003	0.015
\mathcal{F}_{peak_sig}		0.025	0.047	0.045	0.059	0.069	0.053	0.021	0.007	0.056	0.009
K-fact smooth		0.000	0.002	0.002	0.004	0.008	0.015	0.005	0.013	0.023	0.063
dilution		0.011	0.001	0.018	0.006	0.030	0.007	0.019	0.033	0.040	0.101
Total Syst	σ_{syst}	0.053	0.054	0.073	0.074	0.094	0.326	0.281	0.196	0.552	0.296
Stat. uncertainty	σ_{stat}	0.097	0.193	0.293	0.448	0.642	0.894	1.248	1.676	2.227	2.982
Total	σ_{tot}	0.111	0.201	0.302	0.454	0.649	0.952	1.280	1.687	2.294	2.997

TABLE VIII: Breakdown of systematic errors for the RunIIb $\mu K^* K$ mode.

Osc. frequency	ps^{-1}	0.0	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0
Fitted Ampl.	\mathcal{A}	+0.040	+0.135	-0.422	-0.508	+0.815	+0.183	+0.707	+0.540	+0.336	+1.276
$s f_{bkg}$		0.001	0.001	0.001	0.005	0.014	0.016	0.015	0.015	0.016	0.018
$B_s^0 \rightarrow D_s D_s = 4.7\%$		0.027	0.029	0.030	0.025	0.024	0.020	0.024	0.029	0.029	0.019
$B_s^0 \rightarrow D_s \mu \nu X = 5.5\%$		0.019	0.013	0.013	0.010	0.007	0.003	0.007	0.008	0.010	0.005
ΔN_{refl}		0.001	0.010	0.008	0.004	0.004	0.002	0.000	0.004	0.009	0.010
K-factor var. 2%		0.009	0.022	0.022	0.139	0.096	0.066	0.220	0.015	0.113	0.093
$c\tau_{B_s}$		0.001	0.008	0.013	0.013	0.012	0.003	0.010	0.008	0.007	0.011
$p_T^\mu > 6 \text{ GeV}$		0.030	0.024	0.023	0.019	0.016	0.013	0.014	0.020	0.019	0.012
\mathcal{F}_{peak_sig}		0.037	0.094	0.087	0.089	0.122	0.089	0.100	0.090	0.066	0.088
K-fact smooth		0.000	0.001	0.001	0.002	0.005	0.004	0.000	0.001	0.038	0.011
dilution		0.012	0.023	0.006	0.022	0.001	0.059	0.079	0.043	0.006	0.029
Total Syst	σ_{syst}	0.059	0.108	0.099	0.170	0.160	0.129	0.256	0.108	0.143	0.136
Stat. uncertainty	σ_{stat}	0.126	0.247	0.336	0.457	0.586	0.771	0.895	1.187	1.550	1.708
Total	σ_{tot}	0.139	0.270	0.350	0.487	0.608	0.782	0.931	1.192	1.557	1.713

TABLE IX: Breakdown of systematic errors for the RunIIa $\mu\phi\pi$ mode.

Osc. frequency	ps^{-1}	0.0	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0	30.0
Fitted Ampl.	\mathcal{A}	+0.035	-0.099	+0.084	-0.040	-0.399	+0.345	+1.263	+0.902	-0.535	-0.072	+0.140
$c\tau_{Bs}$		0.000	0.001	0.002	0.003	0.002	0.003	0.004	0.002	0.003	0.003	0.008
\mathcal{F}_{peak_sig}		0.000	0.003	0.003	0.004	0.004	0.006	0.010	0.008	0.003	0.006	0.005
$s f_{bkg}$		0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001
\mathcal{F}_{peak_bkg}		0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.003
\mathcal{F}^0		0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.003	0.001	0.003	0.005
$c\tau_{bkg}$		0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.001	0.001	0.000	0.003
σ_{peak_bkg}		0.000	0.003	0.003	0.004	0.004	0.006	0.010	0.008	0.003	0.006	0.005
\mathcal{F}_{tsens}		0.011	0.007	0.003	0.003	0.001	0.000	0.000	0.001	0.004	0.012	0.009
\mathcal{F}_{osc}		0.001	0.003	0.001	0.000	0.000	0.001	0.002	0.004	0.006	0.006	0.010
ΔN_{Ds}		0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.001
ΔN_{refl}		0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001	0.001	0.001	0.001
K-fact var. 2%		0.000	0.005	0.006	0.043	0.071	0.097	0.082	0.266	0.036	0.050	0.153
dilution		0.170	0.153	0.132	0.157	0.148	0.147	0.149	0.033	0.117	0.213	0.181
$B_s^0 \rightarrow D_s D_s = 4.7\%$		0.020	0.016	0.014	0.013	0.011	0.013	0.013	0.009	0.007	0.022	0.023
$B_s^0 \rightarrow D_s \mu\nu X = 5.5\%$		0.016	0.006	0.007	0.004	0.003	0.005	0.006	0.003	0.005	0.014	0.008
K-fact smooth		0.000	0.002	0.002	0.002	0.002	0.004	0.000	0.008	0.023	0.017	0.005
$s f_{sig}$		0.004	0.014	0.015	0.022	0.038	0.030	0.001	0.014	0.078	0.146	0.206
$c\tau_{Bs} = 438\mu \text{ m}$		0.000	0.006	0.010	0.009	0.010	0.014	0.015	0.004	0.012	0.015	0.034
Non-zero $\Delta\Gamma/\Gamma$		0.000	0.000	0.001	0.000	0.001	0.001	0.001	0.004	0.000	0.000	0.001
ϵ versus VPDL		0.000	0.000	0.001	0.001	0.005	0.000	0.020	0.006	0.025	0.001	0.002
$B_s^0 \rightarrow D_s D$		0.011	0.004	0.001	0.011	0.013	0.007	0.001	0.003	0.016	0.028	0.026
Total Syst	σ_{syst}	0.173	0.155	0.135	0.165	0.170	0.180	0.174	0.269	0.151	0.268	0.319
Stat. uncertainty	σ_{stat}	0.036	0.078	0.119	0.176	0.257	0.356	0.484	0.672	0.844	1.045	1.357
Total	σ_{tot}	0.177	0.174	0.180	0.242	0.308	0.399	0.514	0.724	0.857	1.078	1.394

TABLE X: Breakdown of systematic errors for the RunIIb $\mu\phi\pi$ mode.

Osc. frequency	ps^{-1}	0.0	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0	30.0
Fitted Ampl.	\mathcal{A}	+0.060	+0.014	-0.132	+0.059	+0.221	+0.309	+0.650	+0.526	+0.426	+0.882	+0.993
$c\tau_{Bs}$		0.000	0.001	0.002	0.002	0.003	0.003	0.003	0.004	0.002	0.005	0.000
\mathcal{F}_{peak_sig}		0.002	0.003	0.004	0.004	0.004	0.004	0.003	0.000	0.004	0.003	0.005
$s f_{bkg}$		0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001
\mathcal{F}_{peak_bkg}		0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.002
\mathcal{F}^0		0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.001	0.001	0.002	0.003
$c\tau_{bkg}$		0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002
σ_{peak_bkg}		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001
\mathcal{F}_{tsens}		0.013	0.005	0.004	0.001	0.001	0.000	0.002	0.002	0.007	0.010	0.005
\mathcal{F}_{osc}		0.000	0.004	0.003	0.001	0.001	0.001	0.002	0.006	0.008	0.010	0.010
ΔN_{Ds}		0.004	0.003	0.004	0.004	0.004	0.005	0.005	0.008	0.008	0.007	0.006
ΔN_{refl}		0.000	0.003	0.002	0.005	0.006	0.005	0.004	0.002	0.002	0.002	0.001
K-fact var. 2%		0.000	0.014	0.023	0.003	0.002	0.048	0.050	0.119	0.093	0.063	0.035
dilution		0.168	0.136	0.166	0.138	0.200	0.203	0.183	0.225	0.282	0.348	0.327
$B_s^0 \rightarrow D_s D_s = 4.7\%$		0.017	0.019	0.015	0.013	0.013	0.018	0.016	0.009	0.025	0.003	0.002
$B_s^0 \rightarrow D_s \mu\nu X = 5.5\%$		0.018	0.005	0.004	0.005	0.003	0.004	0.008	0.015	0.012	0.011	0.005
K-fact smooth		0.000	0.001	0.002	0.001	0.002	0.007	0.002	0.026	0.024	0.002	0.018
$s f_{sig}$		0.000	0.006	0.009	0.022	0.019	0.043	0.046	0.058	0.039	0.064	0.058
$c\tau_{Bs} = 438\mu \text{ m}$		0.001	0.004	0.006	0.006	0.007	0.007	0.007	0.014	0.009	0.011	0.001
Non-zero $\Delta\Gamma/\Gamma$		0.000	0.000	0.001	0.000	0.001	0.001	0.001	0.002	0.002	0.001	0.005
ϵ versus VPDL		0.000	0.000	0.001	0.000	0.001	0.000	0.013	0.017	0.008	0.002	0.000
$B_s^0 \rightarrow D_s D$		0.007	0.002	0.002	0.002	0.002	0.002	0.002	0.004	0.002	0.015	0.008
Total Syst	σ_{syst}	0.170	0.139	0.169	0.141	0.202	0.214	0.197	0.264	0.303	0.360	0.335
Stat. uncertainty	σ_{stat}	0.044	0.094	0.141	0.202	0.282	0.387	0.509	0.668	0.818	1.023	1.250
Total	σ_{tot}	0.176	0.168	0.220	0.246	0.347	0.443	0.546	0.719	0.872	1.084	1.295

TABLE XI: Breakdown of systematic errors for the RunIIa+b $\pi\phi\pi$ mode.

Osc. frequency	ps^{-1}	0.0	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0
Fitted Ampl.	\mathcal{A}	-0.100	-0.200	-0.300	+0.250	-0.200	-0.850	+1.200	+1.100	+0.150	+1.350
f_{bkg}		0.220	0.000	0.050	0.000	0.000	0.021	0.044	0.047	0.069	0.086
$s f_{sig}$		0.000	0.000	0.100	0.044	0.109	0.071	0.138	0.044	0.108	0.034
2 Asymptotic Gaussians		0.000	0.050	0.000	0.100	0.155	0.071	0.006	0.147	0.050	0.427
ϵ vs. VPDL		0.000	0.000	0.000	0.000	0.055	0.000	0.000	0.000	0.000	0.007
dilution		0.220	0.050	0.050	0.050	0.000	0.092	0.044	0.044	0.039	0.086
Total Syst	σ_{syst}	0.311	0.071	0.122	0.120	0.197	0.138	0.151	0.167	0.143	0.446
Stat. uncertainty	σ_{stat}	0.125	0.225	0.325	0.425	0.550	0.650	0.825	0.900	1.100	1.275
Total	σ_{tot}	0.335	0.236	0.347	0.442	0.584	0.665	0.839	0.915	1.109	1.351

TABLE XII: Breakdown of systematic errors for the RunIIa $\mu K_S K$ mode.

Osc. frequency	ps^{-1}	0.5	3.5	6.5	9.5	12.5	15.5	18.5	21.5	24.5	27.5
Fitted Ampl.	\mathcal{A}	-0.273	+0.518	+0.620	-0.156	-0.293	+0.128	-4.317	-10.338	-9.409	+2.544
$B_s^0 \rightarrow D_s D_s = 4.7\%$		0.005	0.005	0.004	0.004	0.006	0.008	0.006	0.008	0.008	0.014
$B_s^0 \rightarrow D_s \mu \nu X = 5.5\%$		0.066	0.037	0.038	0.051	0.035	0.005	0.009	0.017	0.062	0.091
$p_T^\mu > 6 \text{ GeV}$		0.081	0.045	0.044	0.060	0.041	0.009	0.013	0.012	0.076	0.076
K-fact smooth		0.001	0.005	0.014	0.059	0.003	0.309	0.498	1.023	0.140	0.067
reco K-fact		0.003	0.034	0.074	0.035	0.027	0.748	0.403	1.199	0.734	3.016
K-fact p_T		0.010	0.044	0.044	0.071	0.248	0.123	0.039	5.087	2.187	10.089
$\mathcal{F}_{peak,bkg}$		0.005	0.003	0.008	0.005	0.008	0.044	0.062	0.024	0.106	0.475
$\mathcal{F}_{peak,sig}$		0.010	0.003	0.020	0.043	0.038	0.032	0.152	0.319	0.320	0.310
$c\tau_{B_s}$		0.036	0.053	0.100	0.075	0.009	0.262	0.059	0.718	0.457	3.143
ΔN_{refl}		0.041	0.034	0.090	0.101	0.047	0.187	0.012	0.907	0.510	2.311
ΔN_{D_s}		0.008	0.221	0.036	0.067	0.246	0.510	0.400	0.606	0.537	3.968
$s f_{sig}$		0.035	0.009	0.104	0.279	0.027	0.089	0.494	0.344	0.201	0.441
M_{bkg} shape		0.023	0.082	0.021	0.074	0.147	0.383	0.036	0.322	0.253	2.365
dilution		0.016	0.052	0.060	0.030	0.213	0.588	1.539	0.371	0.223	3.692
\mathcal{F}_{tsens}		0.094	0.037	0.087	0.012	0.055	0.215	0.073	0.358	0.008	2.950
\mathcal{F}_{osc}		0.022	0.048	0.078	0.023	0.059	0.324	0.149	0.365	0.093	2.885
$c\tau_{neg}$		0.024	0.050	0.077	0.028	0.052	0.307	0.090	0.440	0.180	2.884
Cabbibo Ratio		0.026	0.077	0.065	0.009	0.083	0.346	0.155	0.229	0.038	2.986
$D_s D_s$ PDG		0.036	0.062	0.069	0.021	0.061	0.313	0.113	0.384	0.116	2.868
$s f_{bkg}$		0.025	0.045	0.102	0.026	0.135	0.535	0.518	0.061	0.314	3.677
Total Syst	σ_{syst}	0.170	0.295	0.291	0.354	0.485	1.512	1.886	5.584	2.555	14.752
Stat. uncertainty	σ_{stat}	0.408	0.789	1.361	2.204	3.091	4.747	7.535	8.023	11.136	16.586
Total	σ_{tot}	0.442	0.842	1.392	2.232	3.129	4.982	7.767	9.775	11.426	22.197